

WORKSHOP PRIN 2022

BUILDING RESILIENCE TO EMERGING RISKS IN FINANCIAL AND INSURANCE MARKET

In memory of **Anna Rita Bacinello**

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A potential USP approach for demographic risk in Solvency II framework

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Target of our paper



We propose an alternative methodology for assessing capital requirement for idiosyncratic (diversifiable) demographic risk for the main traditional life insurance contracts, where also the relevance of sums insured volatility is put in evidence for risk evaluation.



The proposed formula can represent a possible undertaking-specific approach (USP) in Solvency II framework, also <u>improving the factor-based formula</u> <u>proposed in QIS2 2006</u> (then modified according to a scenario-approach in the final Standard Formula using in practice a stress of BEL).



Numerical analyses are also carried out for some cohorts, to evaluate the goodness of the proposed USP formula using as a benchmark a risk-theory based Partial Internal Model, then confirming how it could be a <u>suitable</u> alternative to Standard Formula or Simulation Models.

Cohort Approach

- We consider a cohort of contracts composed by l_0 policyholders at time 0 with the same characteristics (except for the sums insured).
- We define the r.v. insured sums of the k policyholder at time t as follows:

$$S_{k,t} = s_{k,0} \prod_{\tau=0}^{t-1} \mathbb{I}_{k,\tau}^{L}.$$

where $\mathbb{I}_{k,\tau}^L$ is a Bernoulli r.v. that assumes value equal to one if the policyholder survives from τ to $\tau + 1$.

• Since the cohort is composed by policyholders with the same characteristics (except for the sums insured), we assume that the survival of the policyholders are conditionally independent. Furthermore, we define the sums insured of the whole portfolio as follows:

$$S_t = \sum_{k=1}^{l_0} S_{k,t}$$
 Number of initial cohort's policyholders

Cash-In and Cash-Out

- We consider a vector of cash-flows $\mathbf{X} = \mathbf{X}^{out} \mathbf{X}^{in}$.
- We define the cash-out of the year (i.e. benefits) as follows:

$$X_t^{out} = \sum_{k=1}^{l_0} S_{k,t-1} \cdot \mathbb{I}_{k,t-1}^B$$

where $\mathbb{I}_{k,t-1}^B$ is a dichotomic r.v. which assumes value 1 whereas the k-th policyholders becomes eligible to obtain the benefit in the time span (t-1,t]

• Similarly, we define the cash-in of the year (i.e. premiums) as:

$$X_t^{in} = \sum_{k=1}^{l_0} S_{k,t} \cdot p_t$$
 NOTE: premium rate calculated on a 1st order basis

where p_t is the premium rate per unitary sum insured. Notice that X_t^{in} is \mathcal{F}_t -measurable, as the premium rate is a quantity known when the policies are underwritten.

Best Estimate Liability (BEL)

We define the Best Estimate Liability as follows:

$$\underbrace{R_t} = \sum_{\tau=t+1}^{n} \left(1 + i_t(t,\tau)\right)^{t-\tau} E\left(X_{\tau}^{out}\big|\mathcal{F}_t\right) - \sum_{\tau=t}^{n-1} \left(1 + i_t(t,\tau)\right)^{t-\tau} E\left(X_{\tau}^{in}\big|\mathcal{F}_t\right)$$
BEL = Expectation of discounted parametrization potentials and flower parametrization and specifical parameters are also flower parameters.

where $i_t(t,\tau)$ is the spot rate (EIOPA risk-free rate curve).

perspective net cash-flows (2nd order demogr. bases)

Considering that $S_{k,t}$ is \mathcal{F}_t -measurable and exploiting its definition, it is possible to write

$$R_{t} = S_{t} \cdot \sum_{\tau=t}^{n-1} (1 + i_{t}(t, \tau + 1))^{t-\tau-1} E\left(\sum_{k=1}^{l_{0}} \left(\prod_{s=t}^{\tau-1} \mathbb{I}_{k, s}^{L}\right) \cdot \mathbb{I}_{k, \tau}^{B} \middle| \mathcal{F}_{t}\right)$$

$$- S_{t} \cdot \sum_{\tau=t}^{n-1} (1 + i_{t}(t, \tau))^{t-\tau} E\left(\sum_{k=1}^{l_{0}} \left(\prod_{s=t}^{\tau-1} \mathbb{I}_{k, s}^{L}\right) \cdot p_{\tau} \middle| \mathcal{F}_{t}\right) = S_{t} \cdot \mathcal{R}_{t}$$
BEL rate

where \mathcal{R}_t is the best estimate rate (per unitary sum insured).

depending on insurance type (Term, Endowm., Pure Endowm.)

Annual CDR: relation and decomposition

• Consistently with the existing literature (see Wuthrich and Merz (2013)), we define the Claims Development Result (CDR) between time t and time t+1 as

$$(DR_{t+1}) = (R_t + x_t^{in}) \cdot (1 + i_t(t, t+1)) - X_t^{out} - R_{t+1}$$

$$= CDR_{t+1}^{Idios} + CDR_{t+1}^{Trend} =$$

$$= (R_t + x_t^{in}) \cdot (1 + i_t(t, t+1)) - X_t^{out} - R_{t+1}$$

$$= (R_t + x_t^{in}) \cdot (1 + i_t(t, t+1)) - X_{t+1}^{out} - \hat{R}_{t+1}) + (\hat{R}_{t+1} - R_{t+1})$$

$$= (R_t + x_t^{in}) \cdot (1 + i_t(t, t+1)) - X_{t+1}^{out} - \hat{R}_{t+1}) + (\hat{R}_{t+1} - R_{t+1})$$

$$= (R_t + x_t^{in}) \cdot (1 + i_t(t, t+1)) - X_{t+1}^{out} - \hat{R}_{t+1}) + (\hat{R}_{t+1} - R_{t+1})$$

with

$$\hat{R}_{t+1} = S_{t+1} \cdot \sum_{\tau=t+1}^{n-1} (1 + i_t(t+1, \tau+1))^{t-\tau} E\left(\sum_{k=1}^{l_0} \left(\prod_{s=t+1}^{\tau-1} \mathbb{I}_{k,s}^L\right) \cdot \mathbb{I}_{k,\tau}^B \middle| \mathcal{F}_t\right) \\
- S_{t+1} \cdot \sum_{\tau=t+1}^{n-1} (1 + i_t(t+1, \tau))^{t+1-\tau} E\left(\sum_{k=1}^{l_0} \left(\prod_{s=t+1}^{\tau-1} \mathbb{I}_{k,s}^L\right) \cdot p_\tau \middle| \mathcal{F}_t\right) = S_{t+1} \hat{R}_{t+1}$$

• $i_t(t+1,\tau)$ is the forward rate between t+1 and τ available from the spot curve at time t. \hat{R}_{t+1} and \hat{R}_{t+1} are the best estimate and the best estimate rates of the policyholders in t+1 calculated with demographic basis equal to those used in t, i.e. \mathcal{F}_t . Keep 2nd order demographic bases unchanged from time t to t+1

Idiosyncratic CDR and SaR rate

• It is therefore possible to rewrite CDR_{t+1}^{Idios} in a compact way:

$$\underbrace{CDR_{t+1}^{Idios}} = \sum_{k=1}^{l_0} \left[E\left(S_{k,t} \cdot \left(1 - \mathbb{I}_{k,t}^L\right) \middle| \mathcal{F}_t\right) - E\left(S_{k,t} \cdot \left(1 - \mathbb{I}_{k,t}^L\right) \middle| \mathcal{F}_{t+1}\right) \right] \cdot \underbrace{\left(1 - \mathbb{I}_{k,t}^L\right) \left| \mathcal{F}_{t+1}\right|}_{\text{Sum-at-Risk rate}}$$

Previous relation depends on the SaR rate η_{t+1} connected to the contract. In particular we have:

Positive SaR rate (η)

Negative SaR rate (η)

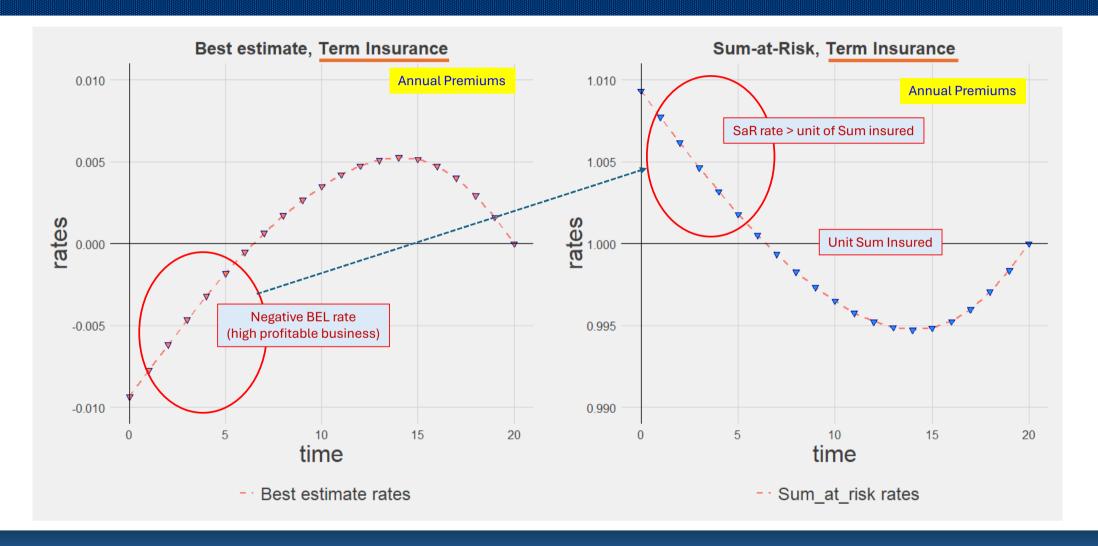
In case of policies that recognize a benefit in case of death (as term insurance. In case of pure endowment contracts and annuity in the deferral period endowment)

$$\underbrace{\eta_{t+1}} = -\sum_{\tau=t+1}^{n-1} \left(1 + i_t(t+1,\tau+1)\right)^{t-\tau} E\left(\sum_{k=1}^{l_0} \left(\prod_{s=t+1}^{\tau-1} \mathbb{I}_{k,s}^L\right) \cdot \mathbb{I}_{k,\tau}^B \middle| \mathcal{F}_t\right) \\
+ \sum_{\tau=t+1}^{n-1} \left(1 + i_t(t+1,\tau)\right)^{t+1-\tau} E\left(\sum_{k=1}^{l_0} \left(\prod_{s=t+1}^{\tau-1} \mathbb{I}_{k,s}^L\right) \cdot p_\tau \middle| \mathcal{F}_t\right)$$

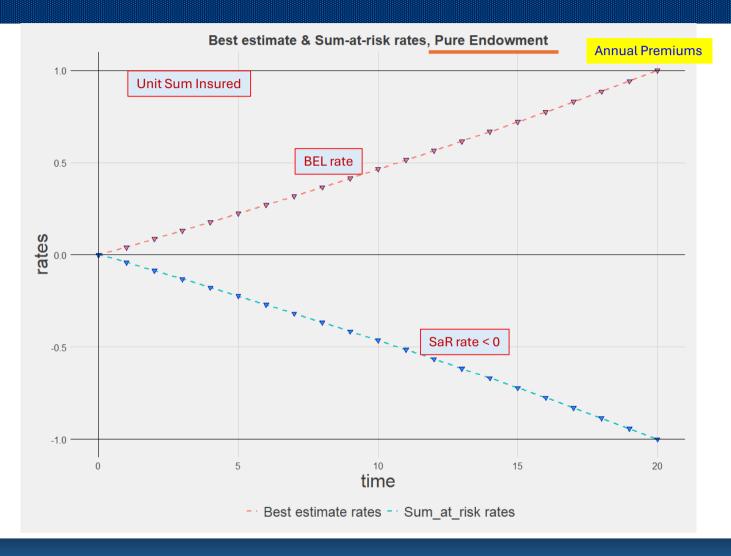
In case of **annuity** in the payment period

$$\underbrace{\eta_{t+1}} = -1 - \sum_{\tau=t+1}^{n-1} (1 + i_t(t+1, \tau+1))^{t-\tau} E\left(\sum_{k=1}^{l_0} \left(\prod_{s=t+1}^{\tau} \mathbb{I}_{k,s}^L\right) \middle| \mathcal{F}_t\right)$$

Term Insurance: BEL rate vs SaR rate



Pure Endowment: BEL rate vs SaR rate



Distribution characteristics of r.v. idiosyncratic CDR

The expected value of the CDR_{t+1}^{Idios} is equal to zero: $E(CDR_{t+1}^{Idios}|\mathcal{F}_t) = 0$.

For the meaning itself of «Best Estimate» (no prudence)

Variance of CDR_{t+1}^{Idios} is obtained as follows:

$$\sigma^{2}\left(CDR_{t+1}^{Idios}\big|\mathcal{F}_{t}\right) = \left(l_{t} \cdot q_{x+t} \cdot (1 - q_{x+t}) \cdot \bar{\mathbf{S}}_{2,t}\right) \cdot E\left(\eta_{t+1}^{2}\big|\mathcal{F}_{t}\right).$$

So the STD(CDR) is depending on the absolute value of SaR rate (increase/decrease according insurance type)

Skewness of CDR_{t+1}^{Idios} is obtained as follows:

Usually by far > 0 unless extreme ages (and decreasing time by time)

Ratio always > 0 and it increases accordingly higher CV of sums insured

$$\gamma \left(CDR_{t+1}^{Idios} \middle| \mathcal{F}_t \right) = \underbrace{-sgn(\eta_{t+1})} \cdot \frac{(1 - 2 \cdot q_{x+t})}{\sqrt{l_t \cdot q_{x+t} \cdot (1 - q_{x+t})}} \cdot \underbrace{\left[\bar{\mathbf{S}}_{3,t} \right]_{3/2}^{\bar{\mathbf{S}}_{3,t}}}_{\mathbf{S}_{4,t}}$$

where $\bar{\mathbf{S}}_{j,t}$ is the j-raw moment of the insured sums at time t.

Then Skew(CDR) is usually: >0 for PureEndowm./Annuity < 0 for Term/Endowm.

NOTE: all these formulae are valid for whatever type of «traditional» life insurance contracts and in case of single/annual premiums

Decreasing time by time for cohort's size l, and increasing by q(1-q) Binomial Std (for not-extreme ages)

Idiosyncratic Mortality Risk (1/2)

- Considering contracts with a positive SaR, we focus on a USP approach for idiosyncratic mortality risk.
- We define the random variable Y_{t+1} as a linear transformation of CDR_{t+1}^{Idios} :

$$Y_{t+1} = -CDR_{t+1}^{Idios} + d$$
 when CDR = $\mathbf{d}
ightarrow \mathbf{Best \, Case}$ = No deaths

where $d = max(CDR_{t+1}^{Idios}) = (R_t + x_t^{in}) \cdot (1 + i_t(t, t+1)) - S_t \hat{\mathcal{R}}_{t+1}$, that is the case in which all policyholders survive at the end of the year.

- We need to make this transformation of the CDR in order to get a **new r.v.** Y_{t+1} with positive skewness and a non-negative support as a LogNormal.
- We define the SCR with the USP approach for mortality as follows:

$$SCR^{USP,m} = VaR_{99.5\%} (Y_{t+1}) - d$$

Idiosyncratic Mortality Risk (2/2)

• Under the assumption that the r.v. Y_{t+1} is LogNormal distributed, we obtain:

$$SCR^{USP,n} = d \cdot \left[\frac{exp\left(\sqrt{ln\left(1 + CV_{Y_{t+1}}^2\right)} \cdot 2.58\right)}{\sqrt{1 + CV_{Y_{t+1}}^2}} - 1 \right]$$

with the coefficient of variation of Y_{t+1} defined as follows:

$$CV_{Y_{t+1}} = \frac{\sqrt{(l_t \cdot q_{x+t} \cdot (1 - q_{x+t}) \cdot \bar{\mathbf{S}}_{2,t}) \cdot E(\eta_{t+1}^2 | \mathcal{F}_t)}}{(R_t + x_t^{in}) \cdot (1 + i_t(t, t+1)) - \dot{R}_{t+1}}.$$

The choice of a <u>LogNormal distribution</u> is made consistently with the underlying assumptions made in SII-Standard Formula for the calibration of many sources of risk (e.g. Premium and Reserve Risk in Non-Life UWRisk)

Idiosyncratic Longevity Risk (Pure Endowment & Annuity-deferral)

• Considering contracts with a negative sum-at-risk (as pure endowment and annuities in the deferral period). We define the random variable W_{t+1} as

$$W_{t+1} = CDR_{t+1}^{Idios} - g$$
 when CDR = $\mathbf{g}
ightharpoonup \mathbf{Worst\ Case}$ = All survive

where $g = min\{CDR_{t+1}^{Idios}\} = (R_t + x_t^{in}) \cdot (1 + i_t(t, t+1)) - S_t \hat{\mathcal{R}}_{t+1}$, that is the case in which all policyholders survive (worst case scenario)

We define the SCR with the USP approach for longevity risk as follows:

$$SCR^{USP,l} = -g \cdot \left[1 - \frac{exp\left(-\sqrt{ln\left(1 + CV_{W_{t+1}}^2\right) \cdot 2.58}\right)}{\sqrt{1 + CV_{W_{t+1}}^2}}\right]$$
NOTE: in case of longevity risk we need only to

where

$$CV_{W_{t+1}} = \frac{\sqrt{\left(l_t \cdot q_{x+t} \cdot (1 - q_{x+t}) \cdot \bar{\mathbf{S}}_{2,t}\right) \cdot E\left(\eta_{t+1}^2 \middle| \mathcal{F}_t\right)}}{\dot{R}_{t+1} - \left(R_t + x_t^{in}\right) \cdot \left(1 + i_t(t, t+1)\right)}.$$

make an additive shift to get non-negative support, (CDR is already positively skewned as the LogNormal)

Idiosyncratic Longevity Risk (Annuity on-payment)

• Considering instead an annuity in payment period, we obtain

$$\underbrace{SCR^{USP,l}} = -g \cdot \left[1 - \frac{exp\left(-\sqrt{ln\left(1 + CV_{W_{t+1}}^2\right)} \cdot 2.58\right)}{\sqrt{1 + CV_{W_{t+1}}^2}}\right]$$

where

$$CV_{W_{t+1}} = \frac{\sqrt{(l_t \cdot q_{x+t} \cdot (1 - q_{x+t}) \cdot \bar{\mathbf{S}}_{2,t}) \cdot E(\eta_{t+1}^2 | \mathcal{F}_t)}}{\dot{R}_{t+1} + S_t - (R_t + x_t^{in}) \cdot (1 + i_t(t, t+1))}.$$

Numerical Analysis: aim and parameters

- We provide here a **comparison of the results (5mln simulations)** obtained by applying the proposed *USP vs Partial Internal Model (PIM)* on some single cohorts .
- The approach has been tested on different types of contract, alternative volatilities of the insured sums and varying portfolio's size.
- In the next table the main parameters of the cohorts are figured out:

Table 1: Model parameters

Characteristics	Value	
Number of policyholders in $t = 0$	15,000	
Cohort age in $t = 0$	40	
Policies duration	20 years	
1st order demographic base	2nd order q_x stressed $\pm 20\%$ (conservative)	
2nd order demographic base	Lee-Carter model applied on 1852-2019 Italy data	
1st order technical rate	1%	
Risk-free curve	August 2023, EIOPA's risk-free curve	
Average sum insured	100,000	
Coeff.Var. of S_0	(2)	

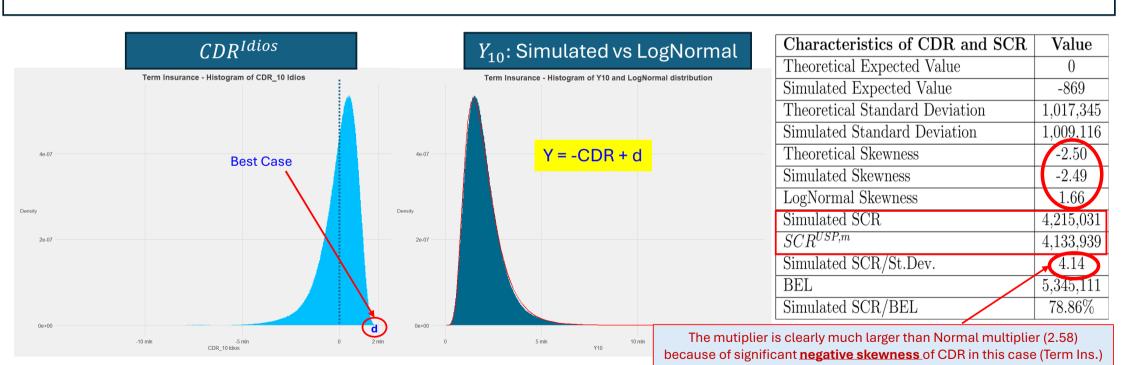
Premiums: Annual and constant

Expenses: not-considered
for simplicity

Please note the reference year is the 10th year from the origin, so assuming to be in t=9 as valuation date.

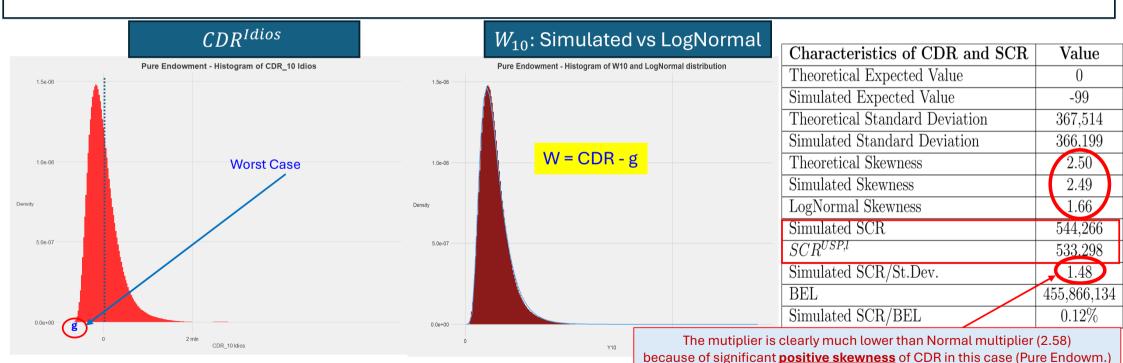
Numerical Analysis: main results – Term Insurance (Year=10)

- Simulated characteristics of CDR show a very good convergence to the theoretical values.
- As expected, CDR distribution in this case is negatively skewed, due to the sign of the SaR rate.
- With the proposed USP approach, we obtain a capital requirement for idiosyncratic demographic risk that is very close to the simulated value provided by the PIM.
- Differences with respect to the PIM results are mainly due to slight differences in skewness and kurtosis



Numerical Analysis: main results - Pure Endowment (Year=10)

- Simulated characteristics of CDR show a very good convergence to the theoretical values.
- As expected, CDR distribution is, in this case positively skewed due to the sign of the SaR rate.
- With the proposed USP approach, we obtain a capital requirement for idiosyncratic demographic risk that is very close to the simulated value provided by the PIM.
- Differences with respect to the PIM results are mainly due to slight differences in skewness and kurtosis



Numerical Analysis: a comparison between 3 products

- We compare here the results obtained for three alternative contracts: Pure Endowment, Term Insurance and Endowment
- It is noticeable the higher volatility and the negative skewness for Term and Endowment insurance contracts
- In all cases, we notice a good proxy of SCR provided by the USP approach

NOTE: similar value to Term Insurance

Pure Endowment

Characteristics	Value
Theoretical Expected Value	0
Simulated Expected Value	-99
Theoretical Standard Deviation	367,514
Simulated Standard Deviation	366,199
Theoretical Skewness	2.50
Simulated Skewness	2.49
LogNormal Skewness	1.66
Simulated SCR	544,266
$SCR^{USP,l}$	533,298
Simulated SCR/St.Dev.	1.48
BEL	455,866,134
Simulated SCR/BEL	0.12%

Term Insurance

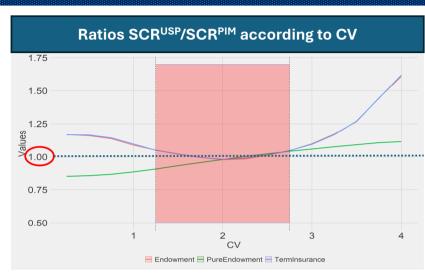
Characteristics	Value
Theoretical Expected Value	0
Simulated Expected Value	-869
Theoretical Standard Deviation	1,017,345
Simulated Standard Deviation	1,009,116
Theoretical Skewness	-2.50
Simulated Skewness	-2.49
LogNormal Skewness	1.66
Simulated SCR	4,215,031
$SCR^{USP,m}$	4,133,939
Simulated SCR/St.Dev.	4.14
BEL	5,345,111
Simulated SCR/BEL	78.86%

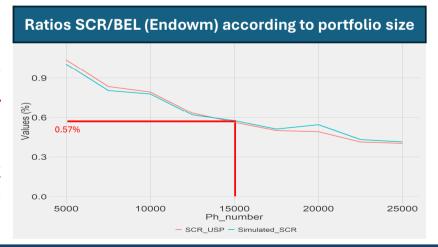
Endowment

Characteristics	Value
Theoretical Expected Value	0
Simulated Expected Value	-421
Theoretical Standard Deviation	646,071
Simulated Standard Deviation	645,644
Theoretical Skewness.	-2.50
Simulated Skewness	-2.49
LogNormal Skewness	1.66
Simulated SCR	2,682,446
$SCR^{USP,m}$	·. 2,625,287
Simulated SCR/St.Dev.	4.15
BEL	467,175,733
Simulated SCR/BEL	0.57%
BEL	4.15 467,175,73

Numerical Analysis: proxy according CV and portfolio size

- We compare here the behaviour of the USP approach varying the sums insured coefficient of variation (CV) from 0 to 4 and the size of the portfolio, respectively.
- The proposed USP approach consistently provides highly reliable estimates of SCR for CVs within a range around 2 (STD(Sums)=200,000 €):
 - for Endowment and Term Insurance, a CV range between 1.25 and 2.75 results in an under/overestimation not exceeding 5%, with substantial overlap between Term and Endowment cases;
 - for Pure Endowments, the CV range moves to 1.75-3.00 being the SCR computed on the short tail of the CDR distribution (which exhibits positive skewness). Similar results are expected also for annuities.
- In case of Endowment type, the size of portfolio shows the diversification effect with a reasonable reduction of the ratio SCR/BEL when the size increases. In all cases we notice a very good approximation assured by the USP approach.
- Clearly, we should keep in mind we have considered only diversifiable risk here (no Trend risk), so this decrease should be rather smoothed in case we are able to add Trend risk.





Trend Risk: a possible algorithm

- 1. Using train data available at time t, fit a projection model to forecast expected mortality rates for the residual coverage period $(q_{x+t}, ..., q_{x+t+n-1})$;
- 2. In each simulation, generate the deaths of the policyholders from l_t Bernoulli r.v.;
- 3. In each simulation h = 1, ...H, build a new train data set DB_{t+1}^h composed by the train data set used at step 1 and by the one-year mortality rates obtained at step 2 in the simulation h.
- 4. In each simulation, re-fit the mortality model selected at step 1 on the new train dataset DB_{t+1}^h , enriched with additional information simulated under real-word probabilities, and estimate new expected mortality rates for the residual coverage period at time t+1.
- 5. Compute for each simulation $CDR_{t+1}^{h,Trend}$;
- 6. Calculate

$$SCR_{Trend} = -min \left[CDR_{t+1}^{h,Trend} : \mathbf{F_{CDR}}_{t+1}^{Trend} \left(CDR_{t+1}^{h,Trend} \right) > 0.5\% \right]$$
(1)

Conclusions and further improvements

- We proposed an alternative methodology for assessing capital requirement for idiosyncratic (diversifiable)
 demographic risk for the main «traditional» types of life insurance contracts, where also the relevance of sums
 insured volatility is put in evidence for risk evaluation.
- The compact formulae here exposed can represent a possible undertaking-specific approach (USP) in Solvency
 II framework, being able to capture the behaviour of random variable CDR of different products based on the specific
 data of the portfolio, split according Cohorts/HRG/Model points.
- Two USP approaches are given for measuring the capital requirement for respectively idiosyncratic mortality and longevity risk.
- Numerical analyses are also carried out for some cohorts, to evaluate the goodness of the proposed approach using
 as a benchmark a risk-theory based Partial Internal Model, then confirming how it could be a suitable alternative
 to Standard Formula or Simulation Models.
- Further analyses may be carried out to check the consistency of our USP approach also for different combinations of duration, sums insured distribution, type of premiums payment.
- Additional studies: in our research we investigated on a model inserting Trend Risk also and to compare the total Demographic risk estimated on the present risk-based approach with the SII - Standard Formula (see Della Corte presentation) and on reinsurance risk mitigation strategies

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