Insurance demand under subjective and objective decision-making approaches

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Insurance demand and premium incentives

We explore the demand for insurance with incentives, using both subjective and objective models.

A risk-averse individual might be willing to pay a sum of money in the present in exchange for compensation in the event of an adverse occurrence in the future. This principle forms the basis for the demand for insurance against damages. The public decision-maker, for its part, might be interested in promoting this demand by reimbursing part of the cost (the insurance premium).

As an example, consider the exposure of agricultural enterprises to the risk of adverse weather events (including in relation to climate change), and the political opportunity to encourage, through premium incentives, the development of an insurance market capable of protecting this productive sector from a portion of potential economic losses.

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Characteristic elements

Some characteristic elements of the insurance demand.

- Insurance premium. Loadings. Deductible.
- Incentives to purchase insurance and implement safety measures.
- Adverse selection. Moral hazard. Status quo bias. Charity hazard.

The insurance demand

Modeling the insurance demand.

We start from the assumption that an adverse event S may occur, and if it happens, an individual (for example, a citizen or a business) incurs a financial loss L. The insurance company estimates that the event S may occur with probability p_C.

The insurance premium

The insurance premium is

$$Q = \mu\lambda p_C L ,$$

obtained by multiplying the expected value $p_C L$ by a coefficient λ that represents the *loadings*, and a coefficient μ that represents the *public incentives*,

$$\lambda > 1, \; 0 \leq \mu \leq 1.$$

The term $\xi = \mu\lambda$ can be defined as the *effective loading*.
Additionally, we denote by p_I the probability of the adverse event according to the individual. In general, this may differ from the probability p_C (estimated by the insurance company).
The individual may also choose to insure only a portion of the loss. The uninsured portion, moreover, can simply not be taken into account, or act as a deductible.

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Subjective models

A mathematical model of insurance demand can be considered *subjective* when it describes the choices of an individual, in the insurance field, taking into account their risk attitudes and economic-financial preferences.
In particular, we present two subjective models, one based on **classical utility** and the other based on **prospect theory.**

Classical Utility

We assume that the individual makes their financial choices based on a utility function U, for which the classic axioms hold:

$$U' > 0\,, U'' < 0$$

(under these assumptions, the individual is risk-averse).
The individual can choose the portion αL of the risk to insure, with

$$0 \leq \alpha \leq 1\,.$$

The expected utility according to the individual is thus

$$E_I[U_\alpha] = (1 - p_I)U(W - \mu\lambda p_C \alpha L) + p_I U(W - \mu\lambda p_C \alpha L - L + \alpha L + \vartheta(1 - \alpha)L)\,.$$

The rational choice is represented by the solution α to the problem

$$\max_\alpha E_I[U_\alpha]\,.$$

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Classical Utility- μ_1 and μ_2 as functions of r

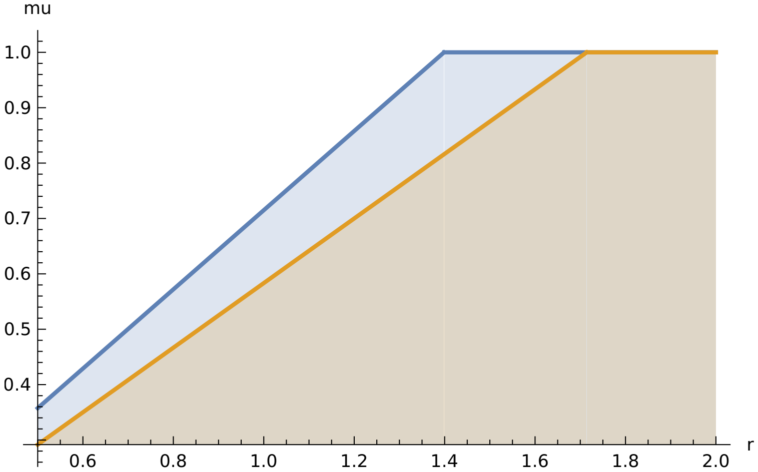

Figure: μ_1 and μ_2 as functions of r *(classical utility).*

Prospect Theory (i)

In prospect theory, individual choices are made based on an evaluation assigned to situations of gain or loss relative to a *reference point*.
Instead of classic utility, a *value function* with the following characteristics is proposed.
The value function v passes through the origin:

$$v(0) = 0\,,$$

is increasing:

$$v'(x) > 0\,,$$

the slope for losses is steeper than for gains *(loss aversion)*:

$$v'(x) < v'(-x)\,,$$

the function is concave in the domain of gains and convex in the domain of losses *(diminishing sensitivity)*:

$$v''(x) \begin{cases} > 0\,, & \text{if } x < 0 \\ < 0\,, & \text{if } x > 0\,. \end{cases}$$

Prospect Theory (ii)

If we assume *full insurance* as the reference point, the individual evaluates deviations from what occurs when they are insured for the entire loss.
In the case $\xi = 1$ (incentives exactly compensating for the loadings), the premium that the individual would pay to insure a part α of the loss is $Q = p\alpha L$, and the variation d with respect to the reference point is

$$d = \begin{cases} p(L - \alpha L)\,, & (1-p) \\ -(1-p)(L - \alpha L)\,, & p\,. \end{cases}$$

Now the average utility (constructed from the value function) results in

$$V = (1-p)v(p(L - \alpha L)) + pv(-(1-p)(L - \alpha L))$$

and the optimal choices α are again solutions to the problem

$$\max_{\alpha} V\,.$$

The model based on prospect theory (with full insurance as a reference point) can be useful for describing individual choices when the premium is perfectly fair. Indeed, in this case according to classical utility the individual would always choose to insure against the entire loss, but it is known that this does not always happen in reality, and to describe such situations one can have recourse to prospect theory.

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Objective models

A mathematical model of insurance demand can be considered *objective* when it guides an individual's choices in insurance based on mathematical evaluations that allow for comparing financial opportunities and risks across different scenarios.
It is observed that, by applying such a criterion, the outcome does not take into account personal preferences. In other words, different individuals applying the same model reach the same conclusion.
Below, we present two objective models, one based on the **mean value criterion** and the other on **the risk index CTE (conditional tail expectation).**

Mean Value Criterion

If an individual has covered the financial risk by paying the insurer the effective premium Q, if the adverse event occurs, they are reimbursed the amount L. For this reason, in any case, the economic outcome for the individual insuring against the total loss is always $X = -Q$ (i.e., they only lose the amount paid as a premium).
On the other hand, the economic outcome for an individual who has not insured can be described by the random variable Y, which is 0 if the adverse event does not occur, and $-L$ if the adverse event occurs.
According to the mean value criterion, the choice to insure is advantageous when

$$E_I[X] > E_I[Y],$$

i.e.,

$$\mu < \frac{p_I}{\lambda p_C}.$$

Thus, from this perspective, insurance is objectively advantageous (like a favorable gamble) when public incentives are sufficiently high (μ small enough). Large loadings (λ high) discourage the choice to insure. Conversely, a high ratio between the individual's probability of the adverse event and that of the insurance company ($\frac{p_I}{p_C}$) favors the choice to insure.

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CTE Index (i)

Now consider the case of insurance with a deductible D. If the estimated loss in the event of an adverse event is represented by the random variable I, the loss for the insured individual is given by the random variable

$$X_I = \begin{cases} I\,, & \text{if } I \leq D \\ D\,, & \text{if } I > D\,. \end{cases}$$

In this context of objective choice based on detailed information, we assume that the random variable I has a continuous and one-to-one distribution function, with a possible jump at 0.
In this case, the effective premium is

$$Q = \xi E[(I - D)_+]\,.$$

As before, ξ indicates the effective loading (which takes into account any public incentives)

$$\xi = \mu\lambda\,.$$

We define *total cost* as the sum of the loss (for the insured) and the effective premium

$$T = X_I + Q\,.$$

CTE Index (ii)

Recall that the *value-at-risk (VaR)* for a random variable Z, with respect to a probability γ, is defined as

$$VaR_Z(\gamma) = \inf\{z : Pr\{Z > z\} \leq \gamma\},$$

and corresponds to the $100(1-\gamma)th$ percentile of Z.
As is known, starting from the VaR, another risk index is defined, the *conditional tail expectation (CTE)*

$$CTE_Z(\gamma) = E[Z|Z \geq VaR_Z(\gamma)].$$

Given a probability γ, we ask what the deductible $\tilde{D}$ is that minimizes the CTE index related to the total cost. In other words, we seek the solution $\tilde{D}$ to the problem

$$CTE_T(\gamma; \tilde{D}) = \min_{D>0} CTE_T(\gamma; D).$$

CTE Index (iii)

In the case

$$\xi > 1$$

(i.e., when public incentives on the premium do not offset the loadings), we set

$$\rho = \xi - 1$$

and

$$\rho^* = \frac{1}{1+\rho}\,.$$

Furthermore, let S_I be the survival function, complementary to 1 of the distribution function for the variable I,

$$S_I(x) = Pr\{I > x\}$$

(in the case of distribution function with a jump at 0, about the inverse function of S, let us say $S_I^{-1}(x) = 0$ for all $S_I(0) \leq x \leq 1$).

It is demonstrated that, under the assumption

$$0 < \gamma < \rho^* < S_I(0)\,,$$

the optimal problem has a unique solution

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CTE Index- D as a function of μ

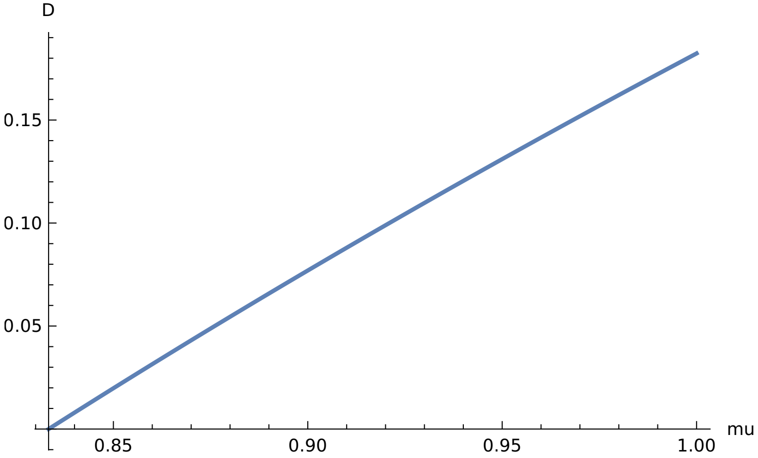

Figure: D as a function of μ *(CTE index)*.

Conclusions

The model based on classical utility theory is suitable for describing the insurance choices of risk-averse individuals, taking into account key elements such as information asymmetries, incentives to purchase insurance and implement safety measures, the damage probability estimated by the insurer or perceived by the individual, and charity hazard. However, when incentives fully compensate for the loadings on the premium, according to classical utility theory, a risk-averse individual would always choose to insure against the total loss. This, however, does not always happen when observing the real behavior of economic agents. In these cases, we saw how the other subjective model, based on prospect theory (with full insurance as the reference point), could be useful, as it allows individuals to choose not to insure even when the premium is perfectly fair.

The model based on expected value shows how insuring is objectively advantageous when premium incentives make the choice akin to a favorable bet. Even when this does not happen (i.e., when premium incentives fail to fully offset the loadings), however, a risk-averse individual may still choose to insure, at least against the part of the loss exceeding a certain deductible, calculating this deductible in a way that minimizes a risk index (as in the case of the model based on the CTE criterion).

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Further research

A possible development of this research is the extension of these choice criteria to a dynamic context. For example, one can ask how having (at least partially) insured or not, and then having suffered an adverse event (loss experience), influences subsequent insurance choices.

In particular, after suffering a loss, the probability p_l that the individual assigns to the adverse event, or his attitude towards risk, may change.

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