An enhanced self-adaptive multi-operator swarm optimization algorithm for ESG-compliant hedge fund

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- Topic:
  building an automated decision support system that can consider the investment preferences of an end-user by combining multi criteria decision analysis and metaheuristics

- Our goal:
  considering a more articulated pre-selection system and introducing a novel metaheuristic for solving long/short portfolio optimization problems

- Methodology:

Objective:	Maximization of the Omega ratio
Constraints:	Cardinality, bound, and budget
Solver:	Adaptive multi-operator particle swarm optimization algorithm (AMPSO)
CHT:	Multi criteria-based expert system for cardinality constraint Repair procedure for bound and budget constraints

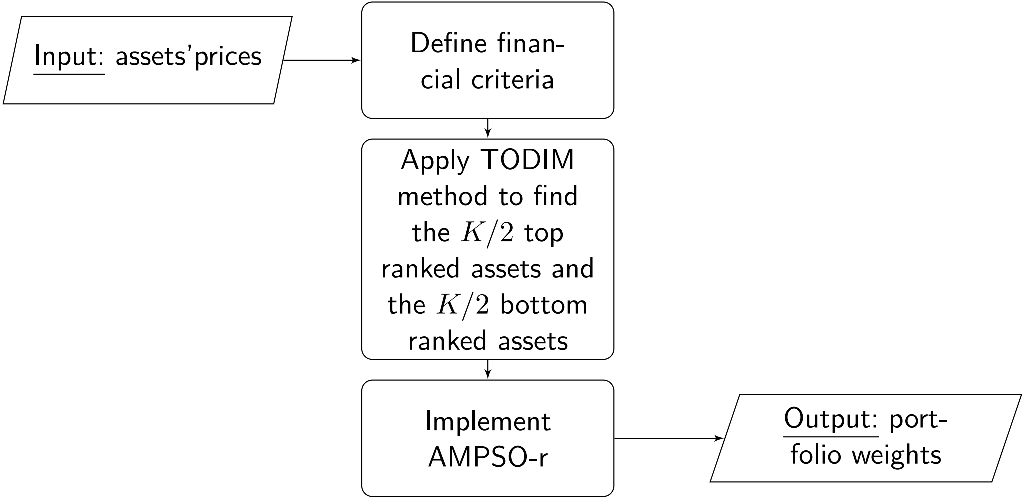
Proposed portfolio optimization model

portfolio weights: $\mathbf{w} \in \mathbb{R}^n$
asset returns in scenario j: $\mathbf{r}^{(j)} \in \mathbb{R}^n$
portfolio return in scenario j: $R_p^{(j)} = \mathbf{w}^\top \mathbf{r}^{(j)}$

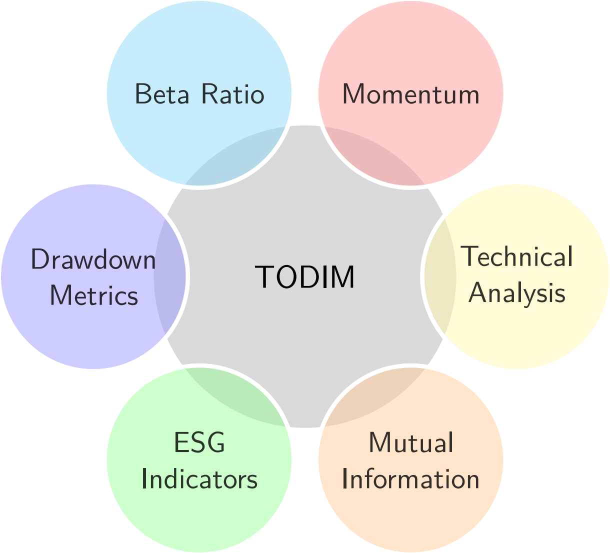
leverage value: $s \in (0, 1)$

$$
\begin{aligned}
\underset{\mathbf{w} \in \mathbb{R}^{\mathbf{n}}}{\text{maximize}} \quad & \Omega(\mathbf{w}) = \frac{\sum_{j=1}^{T} \max(R_p^{(j)} - L, 0)}{\sum_{j=1}^{T} \max(L - R_p^{(j)}, 0)} \\
\text{s.t.} \quad & \sum_{i=1}^{n} w_i = 1 \\
& \sum_{i=1}^{n} \delta_i = K \\
& -s \le \delta_i w_i \le (1 + s), \quad \forall i
\end{aligned}
$$

Knowledge-based financial management system



Stock screening module



Market phase and volatility type calculation

- Tools employed for market phase: moving averages (MA), average directional index (ADX), recent returns

Bull: if $\Delta MA >$ threshold $\wedge\ recentReturn > 0 \wedge ADX > 20$

Bear: if $\Delta MA <$ −threshold $\wedge\ recentReturn < 0 \wedge ADX > 20$

Sideways: otherwise

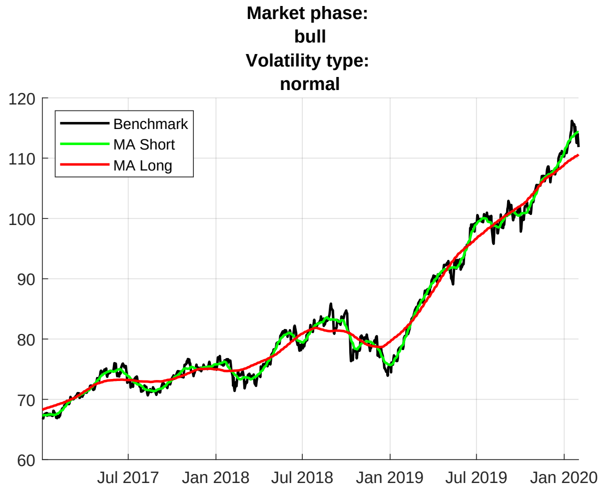
- Tools employed for volatility type: $volRatio = \frac{\text{volatility of the last 20 days}}{\text{volatility of all available days}}$

High: if $volRatio > 1.2$

Low: if $volRatio < 0.8$

Normal: otherwise

Market phase and volatility type calculation



Phase-independent criteria for stock screening

- **Momentum** measure to exploit the ability of individual stocks to generate value over time
  - $MOM_i(t_1, t_N) = \prod_{t=t_0}^{t_{N-1}} (1 + r_{i,t}) - 1$
- **Upside-to-downside beta ratio** to assess the responsiveness of a stock with respect to upward and downward market movements
  - U/D Ratio$_i = \frac{\beta_i^+}{\beta_i^-}$
    where $\beta^- = \frac{Cov(R_i, R_B | R_B < \tau)}{Var(R_B | R_B < \tau)}$ and $\beta^+ = \frac{Cov(R_i, R_B | R_B > \tau)}{Var(R_B | R_B > \tau)}$
- **Drawdown-Based metrics** to evaluate the risk-adjusted quality of asset performance
  - Maximum drawdown (MaxDD) measures the largest peak-to-trough decline in price and indicates worst-case loss
  - Recovery factor measures how efficiently an asset recovers from losses
    $RF = \frac{\text{Final value - Initial value}}{|MaxDD|}$
  - Ulcer performance index $UPI = \frac{\text{average return}}{\text{Ulcer index}}$

Phase-independent criteria for stock screening

- **ESG indicators** allow for the integration of non-financial performance indicators that may reflect long-term sustainability and risk exposure
  - *ESG momentum* captures the direction and speed of ESG score improvements, signaling firms that are actively enhancing their sustainability profile
    - change in ESG score over a specified horizon (e.g., 1, 3, 6, or 12 months)
  - *ESG volatility* reflects the stability of ESG scores over time, identifying firms with consistent ESG practices and lower reputational or regulatory risk
    - standard deviation of ESG scores over a longer horizon (e.g., 18, 24, 30, or 36 months)
- Empirical justification (Magnani, Guidolin, and Berk (2024))
  - ESG momentum is shown to be a systematic risk factor
    - short-term improvements in ESG scores can predict lower cost of equity and generate alpha
  - ESG volatility is associated with lower uncertainty and higher risk-adjusted returns
    - portfolios long on stable ESG firms and short on volatile ones outperform

Phase-dependent criteria for stock screening

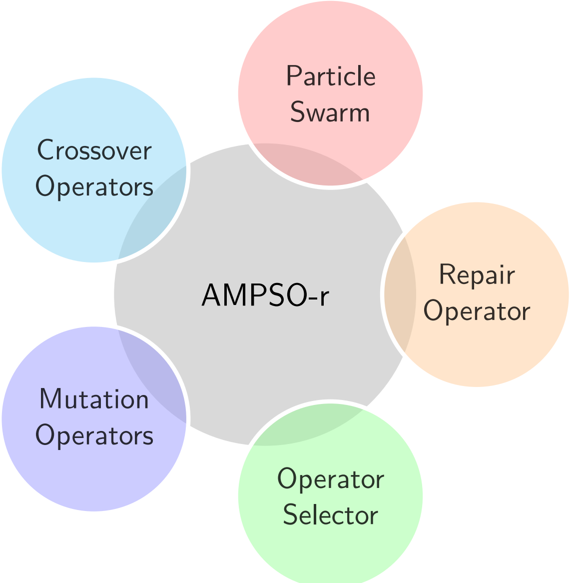
- **Mutual information** captures both linear and non-linear relationships between an asset and a benchmark, making it particularly useful when traditional correlation may fail to detect complex dependencies
  - ⇝ let X, Y two random variables, then $MI(X,Y) = \sum_{x\in\mathcal{X}} \sum_{y\in\mathcal{Y}} p(x,y) \log_2 \left(\frac{p(x,y)}{p(x)p(y)}\right)$
  - ⇝ Adjust scores based on market phase:
    - ▷ Bull ⇒ prefer high MI (strong co-movement)
    - ▷ Bear ⇒ prefer low MI (diversification)
    - ▷ Sideways ⇒ prefer MI near 0.5 (moderate, stable linkage)

- Signals from **technical analysis** depending on the market context

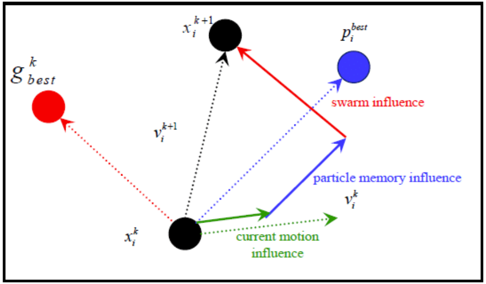
Market Phase	Volatility	Signal Function
Bull	High / Normal	`bullSignal`
Bear	High / Normal	`bearSignal`
Sideways	High	`sidewaysHighVolSignal`
Sideways	Low / Normal	`sidewaysLowVolSignal`

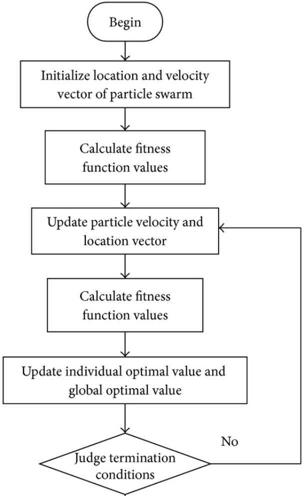
TODIM method - comparisons and rankings

1. *Constructing the multi-criteria decision making matrix* $A = (a_{i,j})_{m \times s}$
2. *Binning criteria matrix* A'
3. *Normalizing the binned matrix*
   - $\rightsquigarrow$ $N'_{i,j} = \frac{a'_{i,j} - \min_i a'_{i,j}}{\max_i a'_{i,j} - \min_i a'_{i,j}}$ for benefits and $N'_{i,j} = \frac{\max_i a'_{i,j} - a'_{i,j}}{\max_i a'_{i,j} - \min_i a'_{i,j}}$ for costs
4. *Computing alternative comparisons* for criterion c_j of alternative a_i against alternative a_k
   - $\rightsquigarrow$ $CS_j(a_i, a_k) = \begin{cases} \beta_j \left(N'_{i,j} - N'_{k,j}\right)^{\eta_1} & \text{if } N'_{i,j} \geq N'_{k,j} \\ -\xi \beta_j \left(N'_{k,j} - N'_{i,j}\right)^{\eta_2} & \text{if } N'_{i,j} < N'_{k,j} \end{cases}$
5. *Calculating the final comparison score concerning each criterion*
   - $\rightsquigarrow$ $FS_j(a_i) = \sum_{k=1}^{m} CS_j(a_i, a_k)$
6. *Determining the final ranking between alternatives*
   - $\rightsquigarrow$ $\mathcal{R}(a_i) = \sum_{j=1}^{s} FS_j(a_i)$


Particle
Swarm
Crossover
Operators
AMPSO-r
Repair
Operator
Mutation
Operators
Operator
Selector

- Are distributed behavioral procedures
- Mimick the movements of a bird flock or a fish schooling that searches for food





Variation operators (1/3)

are applied after standard PSO updates

1. **Arithmetic Crossover (PSO-AX)**
   Combines two randomly selected particles using a weighted average of their positions. Velocities are updated proportionally
2. **Novel Multi-Parent Crossover (PSO-NMPCO)**
   Recombines three randomly selected particles using normalized random weights. If the offspring outperforms the parent, it replaces it
3. **Blend Crossover (PSO-BLX-α)**
   Uses the BLX-α operator to generate offspring within an extended range between two parents. Parent selection is based on roulette wheel selection
4. **Parent-Centric Crossover (PSO-PCX)**
   Generates offspring around a selected parent and the centroid of other parents using Gaussian perturbations

5. **Randomized Parent-Centric Crossover (PSO-PCX$_r$, PSO-PCX$_r^*$)**
   A variation of PCX where the parent to be mutated is selected randomly to enhance exploration. PSO-PCX$_r^*$ includes checks to ensure parent diversity.
6. **Discrete Crossover (PSO-DX)**
   Applies discrete crossover between a particle's new position and:
   - its personal best (PSO-DX$_y$),
   - the global best (PSO-DX$_{\hat{y}}$),
   - or a weighted combination of both (PSO-DX$_{y\hat{y}}$)

   Implemented with either one-point or uniform recombination
7. **Global Best-Centric Crossover (PSO-PCX$_{\hat{y}}$, PSO-PCX$_{\hat{y}}^*$)**
   A PCX variant where the global best is always the mutated parent.
   PSO-PCX$_{\hat{y}}^*$ ensures parent diversity before applying crossover

Variation operators (3/3)

9. **Parent-Centric Crossover with Generalized Generation Gap (PSO-PSPG)**
   Applies PCX asynchronously with a crossover probability p_c. If not applied, standard PSO with constriction coefficient is used. The best individuals among parents and offspring are retained
10. **Gaussian Mutation**
    Introduces stochastic perturbations to particle positions or velocities by sampling from a Gaussian distribution

Objective: dynamically manage the trade-off between exploration and exploitation during the execution of the algorithm

General Functioning:

- At each iteration, the controller selects the recombination operator to apply
- The selection is based on the historical performance of each operator with respect to:
  - *Population quality* (mean fitness)
  - *Population diversity* (entropy)

Main Components:

1. **Aggregated Criteria Computation:** tracks changes in fitness and entropy
2. **Reward Computation:** assigns a reward to each operator based on its impact
3. **Credit Assignment:** aggregates rewards over time
4. **Operator Selection:** chooses the next operator based on credit scores

Dealing with box and budget constraints

- Let $C \subseteq \mathbb{R}^K$ be given by

$$C = \Big\{\mathbf{w}^* \in \mathbb{R}^K : \mathbf{1}^\top \mathbf{w}^* = 1,\, 0 \le w_k \le 1 + s \text{ for } k \in \{1, \ldots, n_{\text{long}}\},$$
$$-s \le w_k \le 0 \text{ for } k \in \{n_{\text{long}} + 1, \ldots, K\}$$

where $\mathbf{w}^* \in \mathbb{R}^K$, with the first n_{long} components being the long leg and the last n_{short} being the short leg

- **Projection onto the intersection of the hyperplane and the box**

$$P_C(\mathbf{w}^*) = P_{\text{Box(s)}}\left(\mathbf{w}^* - \mu^* \mathbf{1}\right)$$

where μ^* is a solution of the equation

$$\mathbf{1}^\top P_{\text{Box(s)}}(\mathbf{w}^* - \mu \mathbf{1}) = 1$$

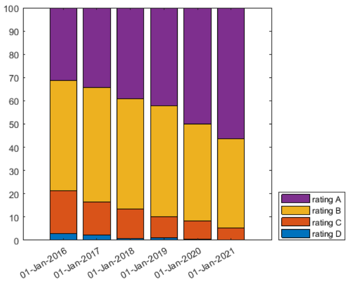
and $\text{Box(s)} = \Big\{\mathbf{y} \in \mathbb{R}^K : 0 \le w_k \le 1 + s \text{ for } k \in \{1, \ldots, n_{\text{long}}\}$ and $-s \le w_k \le 0 \text{ for } k \in \{n_{\text{long}} + 1, \ldots, K\}\Big\}$

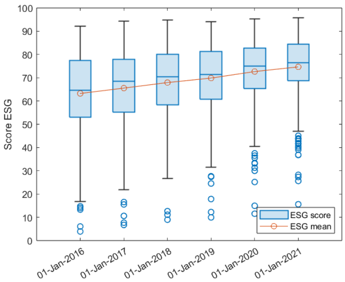
- Data type: daily closing prices and Refinitiv's ESG scores

Data set name	n stocks	Time window
STOXX Europe 600	435	$01/07/2013 - 28/02/2020$

- Refinitiv's ESG scores
  - ⇝ are presented as percentile rankings, with 100 representing the highest score and 0 the lowest
  - ⇝ reflect the relative performance of ESG factors within the company's sector (for environmental and social aspects) and country of incorporation (for governance) and are updated monthly
- The market value-weighted index of the 435 stocks included in the investment universe as the proxy for the market

Analysis of ESG statistics

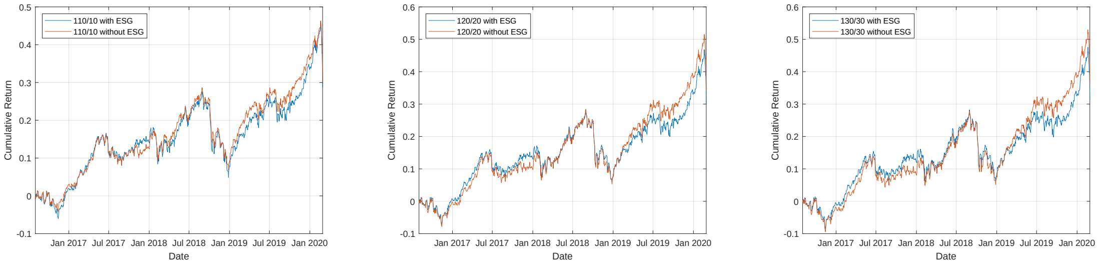




- The binding request of compliance to certain standards and the recent regulations have led to substantial ESG score improvements for securities in the European market over time
- Relying solely on ESG scores may lose its effectiveness as a tool for promoting the sustainability principles among financial actors
- Identifying portfolio allocations in assets that have a higher sustainability growth over time, even if with lower ESG scores, could be a more compliant strategy to leverage ESG information in allocation decisions

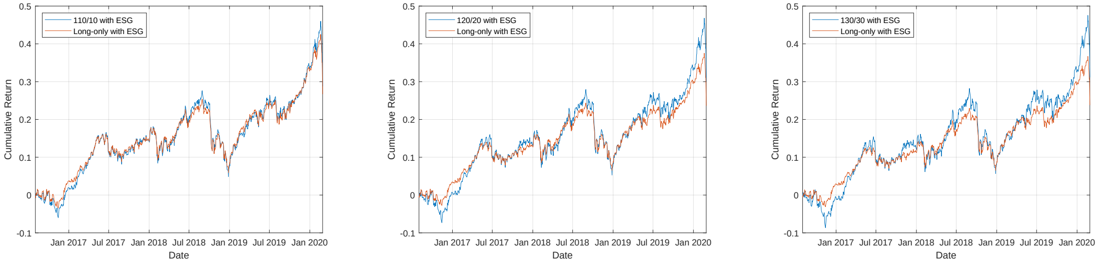
- Monthly rebalancing, out-of-sample window of 44 months from 29/07/2016 to 28/02/2020
- Cardinality parameter $K = 86$ (corresponding to 20% of the investment universe) with $n_{\text{long}} = n_{\text{short}} = 43$
- Leverage values $s \in \{0.10, 0.20, 0.30\}$
- Equal weighting scheme
  - ⇝ long leg: $w_{\text{long},i} = \frac{1+s}{n_{\text{long}}}$
  - ⇝ short leg: $w_{\text{short},j} = -\frac{s}{n_{\text{short}}}$
- No consideration given to transaction or margin costs (maintenance margins, interest payments)

Long/Short with ESG vs. Long/Short without ESG



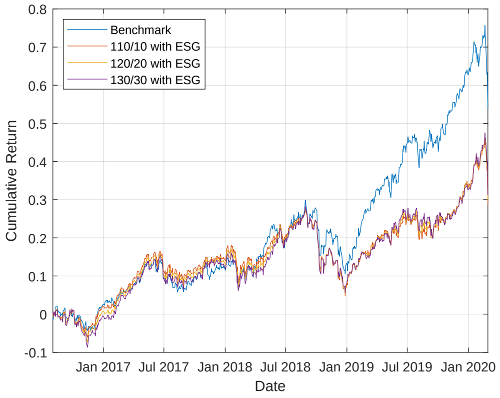
- Long/short strategies based on ESG criteria exhibit superior performance in 2017, regardless of the leverage level
- In 2018, long/short strategies with and without ESG integration exhibit comparable performance
- Over the past two years, strategies based solely on financial criteria have outperformed those incorporating ESG considerations

Long/Short with ESG vs. Long-only with ESG



- To compare performance under equal net exposure, we select the top $n_{\text{long-only}} = \lfloor \frac{n_{\text{long}}}{1+s} \rfloor$ stocks and assign them equal weights $w_{long-only,i} = \frac{1}{n_{\text{long-only}}}$, setting all other weights to zero
- Leveraged strategies that incorporate ESG criteria still outperform their long-only counterparts
- Higher leverage amplifies differences in return peaks, whereas drawdowns remain broadly similar, with the exception of Q1 2017

Long/Short with ESG vs. Benchmark



- The long/short strategies are highly correlated; in particular, since July 2018, they have produced virtually identical ex-post results in terms of cumulative returns
- Until September 2018, they closely tracked the benchmark's behavior using 20% of its constituents, and often managed to outperform it
- After September 2018, the selection criteria have provided underperforming signals at the aggregate level

Conclusions and future research

- By using only the MCDM module for stock selection and adopting a completely uninformed weighting scheme, the trading system is able to generate value over time
- The study will now focus on:
  1. implementing the developed metaheuristic to solve the Omega ratio maximization problem under long/short constraints
  2. investigating the predictive capabilities of the considered criteria/classifiers
  3. exploring alternative weighting methods for the TODIM approach
  4. analyzing the strategy's sensitivity to portfolio cardinality and assessing the potential impact of transaction costs on ex-post performance

Thank you for your attention