Multi-index parametric insurance for agricultural weather risk management

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Sapienza University of Rome - MEMOTEF

Workshop Prin 2022

Building resilience to emerging risk in financial and insurance markets

Diamante (CS), June 12th, 2025

Finanziato dall'Unione europea NextGenerationEU

Ministero dell'Università e della Ricerca

Italiadomani PIANO NAZIONALE DI RIPRESA E RESILIENZA

SAPIENZA UNIVERSITÀ DI ROMA

Agenda

1. Motivation
2. Weather multi-index insurance contract
3. Numerical results
4. Concluding remarks

1. **Motivation**
2. Weather multi-index insurance contract
3. Numerical results
4. Concluding remarks

Increasing frequency and severity of extreme weather events underscore the need for innovative risk management tools

In Italy [**Legambiente-Unipol Report, 2023**]:

- **378 extreme weather events**, marking a **22%** increase from the previous year:
  118 flooding and flash floods due to intense rainfall, **82** cases of damage from tornadoes and strong winds, **39** hailstorm damage, **35** river overflows, **26** storm surges, **21** cases of damage from prolonged drought, **20** instances of extreme urban temperatures, **18** landslides triggered by heavy rain

- extremely **high** temperatures:
  July 20th, **43°C** in Olbia (Sardinia); October 1st, **41,8°C** in Empoli and **33°C** in Florence Peretola (Tuscany); October 30th, **33°C** in Palermo (Sicily)

- **52** hailstorms in **one day** (July 19th, Veneto)

Agriculture is highly climate-sensitive, especially in developing regions, since unpredictable weather events (droughts, floods, irregular rainfall) might devastate yields and plunge farming households into poverty

- **May 2023 floods in Emilia-Romagna** affected nearly 21000 farms; estimated agricultural losses exceed **€1.5 billion** (Ravenna, Forlì-Cesena, Rimini, and Bologna provinces)
- 2023 regional **Gross Marketable Production** (GMP) dropped significantly:
  - Fruit orchards: **–28.6%**
  - Cereals: **–30.1%**
  - Industrial crops: **–16.5%**
  - Total crop production: **–17%**
- **EU Solidarity Fund** (Decision 2024/2772): **€378.8 million** allocated to Italy for this disaster (certified damages to €8.5 billion)

- **[PERILS, 2024]** final report: only €495 million in insured losses, i.e. reflects low flood coverage among households and SMEs. About 50% of total losses involved public infrastructure, typically uninsured.

- Despite causing the highest economic losses, the May floods had a moderate insurance impact. By contrast, July convective storms in Northern Italy led to record insured losses of €4.8 billion (CRESTA CLIX), due to broader coverage for hail, wind, and rain.

Solution: **Multi-index** parametric insurance as a promising tool to manage the **complex, multidimensional** nature of weather-related agricultural risks and overcome

(i) **Basis Risk:** Payouts may not match actual losses if the index (e.g., rainfall at a weather station) does not reflect conditions on the farm

(ii) **Limited Risk Capture:** Single indices fail to represent complex interactions that drive agricultural losses (drought + heatwaves, excess rain + low temperatures)

Idea: Given n potential adverse weather events,

- construct an insurance contract that provides protection against their occurrence **simultaneously**
- The contract triggers a payout if **at least one** of these adverse events takes place

- Let $X_1, X_2, \dots, X_n$ be independent random variables representing n distinct weather indicators (e.g., rainfall, temperature, wind speed, etc.)
- For each variable X_i, define a reference set I_i (discrete or continuous) identifying the range of acceptable values
- A weather event is classified as **adverse** (and triggers coverage) if the realized value of the corresponding indicator falls **outside** its reference set:

$$X_i \notin I_i, \ i = 1, \dots, n \tag{1}$$

- Define $L := L(X_1, \dots, X_n)$ as a **loss function**, which specifies the payout structure in response to the joint realization of the weather indicators.

Case 1 **Fixed payout**

$$L = L(X_1, \ldots, X_n) = K \cdot \mathbf{1}_{\{\bigcup_{i=1}^{n}(X_i \notin I_i)\}} = K \cdot \left[1 - \prod_{i=1}^{n} \mathbf{1}_{\{X_i \in I_i\}} \right] , \qquad (2)$$

where $K \in \mathbb{R}^+$ is a pre-determined amount

Case 2 **Variable payout**

$$L = L(X_1, \ldots, X_n) = \max\left\{ c_1 (X_1 - q_1)^+ , \ldots, c_n (X_n - q_n)^+ \right\} , \qquad (3)$$

where c_i, $i = 1, \ldots, n$, is the **tick**, q_i, $i = 1, \ldots, n$, **quantile** associated to the CDF of X_i

Case 3 **Variable payout, minimum guarantee, coverage cap**

$$L = L(X_1, \ldots, X_n) = m \left(1 - \prod_{i=1}^{n} \mathbf{1}_{\{X_i < q_i\}} \right) + \min\left\{ M, \max\left\{ c_1 (X_1 - q_1)^+ , \ldots, c_n (X_n - q_n)^+ \right\} \right\} , \qquad (4)$$

where c_i, $i = 1, \ldots, n$, is the **tick**, q_i, $i = 1, \ldots, n$, is the **quantile** associated to the CDF of X_i, m is the **minimum guarantee**, M is the **coverage cap**

Key Question: What is the fair premium the policyholder should pay for coverage?

- **Fixed Payout Case:** Let $\lambda \in \mathbb{R}^+$ be the **loading factor** and K the **fixed indemnity**. The premium is

$$Q = \lambda\, \mathbb{E}[L] = \lambda\, [pK + (1-p) \cdot 0] = \lambda p K, \tag{5}$$

where $p = \mathbb{P}\left(\bigcup_{i=1}^{n} \{X_i \notin I_i\}\right)$ is the probability of at least one adverse event

- **General Case: No closed-form** expression in general, **but** the expected loss satisfies the inequality

$$\mathbb{E}[L] \leq \sum_{i=1}^{n} \mathbb{E}[L_i], \tag{6}$$

where $L_i := L(X_i)$ denotes the **marginal loss** associated with the i-th weather variable

Pricing multi-index parametric insurance requires **estimating the probability of adverse weather events across multiple variables** (Burn analysis, **[Taib and Benth, 2012]**)

- Consider time series of n weather variables, each of length N
- Fix confidence level α and define the trigger threshold as the empirical α-quantile $\hat{q}_\alpha$
- For each time period $k = 1, \ldots, N$, define the **adverse event indicator** as

$$T_k := 1 - \prod_{i=1}^{n} \mathbf{1}_{\{X_i^{(k)} \leq \hat{q}_\alpha\}} \tag{7}$$

- Estimate the **trigger probability** via burn analysis

$$\hat{p}_{\text{burn}} = \frac{1}{N} \sum_{k=1}^{N} T_k \tag{8}$$

- The premium with **safety loading** θ is

$$Q_{\text{burn}} = (1 + \theta)\, K\, \hat{p}_{\text{burn}} \tag{9}$$

Alternatively, we use **copulas** to model the joint distribution of weather variables (**[Bokusheva, 2018], [Bressan and Romagnoli, 2021]**)

- A copula C is a multivariate distribution on $[0,1]^n$ with uniform marginals

$$F_i(x_i) = \mathbb{P}(X_i \leq x_i), \quad i = 1, \ldots, n, \tag{10}$$

so that the joint CDF satisfies **Sklar's theorem**

$$F(x_1, \ldots, x_n) = C\left(F_1(x_1), \ldots, F_n(x_n)\right) \tag{11}$$

or, equivalently,

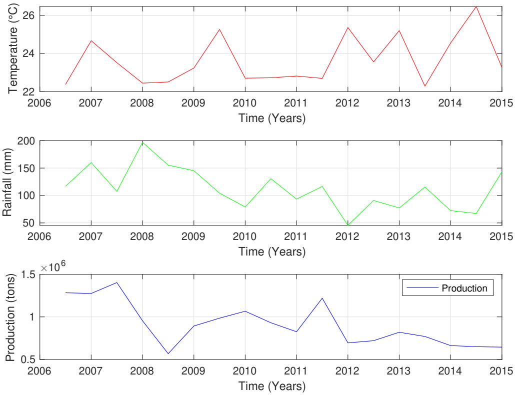
$$C(u_1, \ldots, u_n) = F\left(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)\right). \tag{12}$$

- **Student t copulas:** elliptical, symmetric, parametric form with tail dependence (degrees of freedom ν and correlation matrix Σ)
- **Estimated probability** of adverse event

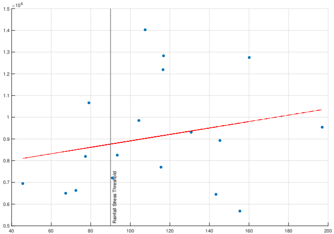
$$\hat{p}_{cop} = 1 - \sum_{k=0}^{2^n-1} (-1)^{|s_k|} C\left(u_1^{(k)}, \ldots, u_n^{(k)}\right) \tag{13}$$

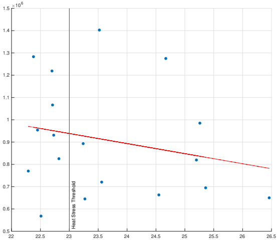
Example: $n = 2$

$$\hat{p}_{cop} = 1 - [C(F_1(b_1), F_2(b_2)) - C(F_1(a_1), F_2(b_2)) - C(F_1(b_1), F_2(a_2)) + C(F_1(a_1), F_2(a_2))] \tag{14}$$



- 1984–2023 time series for **durum wheat** in Bari (Italy) to explore
  - **March–April rainfall**, **June temperatures** (NASA website)
  - **durum wheat production** (2006–2023, ISTAT)





- **Low March–April rainfall** and **High June temperatures** lead to reduced yields
- **Risk indicators:**
  - X_1: Cumulative precipitation (March–April), $\mu_1 = 94.56$, $\sigma_1 = 34.29$
  - X_2: Residuals from linear trend of June temperatures, $\mu_2 = 0$, $\sigma_2 = 1.11$
- **Reference thresholds:**
  - $I_1 = (60.26, +\infty)$ mm; 7 of 40 years below this
  - $I_2 = (-\infty, 1.11)$°C; 8 of 40 years above this
  - **Union of adverse events:** 12 of 40 years; $\hat{p} = 0.3$

Contractual form	*Parameters*	*Premium (empirical)*	*Premium (copulas)*
Fixed p.o.	$\lambda = 1.1$ $K = 1$	$Q_{burn} = 0.330$	$Q_{cop} = 0.274$
Variable p.o.	$\lambda = 1.1$ $c_1 = \sigma_1^{-1},\ c_2 = \sigma_2^{-1}$	$Q_{burn} = 0.140$	$Q_{cop} = 0.115$
Variable p.o., min g	$\lambda = 1.1$ $c_1 = \sigma_1^{-1}, c_2 = \sigma_2^{-1}$	$m_{burn} = 0.330$ $Q_{burn} = 0.249$	$m_{cop} = 0.274$ $Q_{cop} = 0.190$
Variable p.o., min g, max lim	$M = 2, \lambda = 1.1$ $c_1 = \sigma_1^{-1}, c_2 = \sigma_2^{-1}$	$m_{burn} = 0.330$ $Q_{burn} = 0.249$	$m_{cop} = 0.274$ $Q_{cop} = 0.187$

Panel A: Empirical analysis			
Contractual form	*Parameters*	*Premium*	*Sum of premia*
Fixed p.o.	$\lambda = 1.1, K = 1$	$Q = 0.330$	$Q_1 + Q_2 = 0.4125$
Variable p.o.	$\lambda = 1.1,$ $c_1 = \frac{1}{\sigma_1}, c_2 = \frac{1}{\sigma_2}$	$Q = 0.140$	$Q_1 + Q_2 = 0.158$
Variable p.o., min g	$\lambda = 1.1, c_1 = \frac{1}{\sigma_1},$ $c_2 = \frac{1}{\sigma_2}$	$m = 0.330$ $Q = 0.249$	$m = 0.330$ $Q_1 + Q_2 = 0.294$
Variable p.o., min g., max lim	$\lambda = 1.1,$ $c_1 = \frac{1}{\sigma_1}, c_2 = \frac{1}{\sigma_2},$ $M = 2$	$m = 0.330$ $Q = 0.249$	$m = 0.330$ $Q_1 + Q_2 = 0.294$
Panel B: Student-t Copula			
Contractual form	*Parameters*	*Premium*	*Sum of premia*
Fixed p.o.	$\lambda = 1.1, K = 1$	$Q = 0.274$	$Q_1 + Q_2 = 0.317$
Variable p.o.	$\lambda = 1.1,$ $c_1 = \frac{1}{\sigma_1}, c_2 = \frac{1}{\sigma_2}$	$Q = 0.115$	$Q_1 + Q_2 = 0.132$
Variable p.o., min g	$\lambda = 1.1, c_1 = \frac{1}{\sigma_1},$ $c_2 = \frac{1}{\sigma_2}$	$m = 0.330$ $Q = 0.190$	$m = 0.330$ $Q_1 + Q_2 = 0.2185$
Variable p.o., min g., max lim	$\lambda = 1.1,$ $c_1 = \frac{1}{\sigma_1}, c_2 = \frac{1}{\sigma_2},$ $M = 2$	$m = 0.330$ $Q = 0.187$	$m = 0.330$ $Q_1 + Q_2 = 0.212$

Example 1: Risk Diversification Across n i.i.d. Policies

Fixed payout K, estimated adverse event probability $\hat{p} = 0.25$

How many policies are needed to apply probabilistic loadings of c% while ensuring a ruin probability below a fixed confidence level?

Remark We assume i.i.d. risks, consistent with the insurer's diversification approach

	$c = 0.1$	$c = 0.15$	$c = 0.2$
$\alpha = 90\%$	493	219	124
$\alpha = 95\%$	806	359	202
$\alpha = 99\%$	1607	715	402

Reserve: $R = W + n \cdot Q_i - X$, where $X = \sum_{i=1}^{n} Y_i$, $Y_i \sim Ber(p)$ with value K with probability p and 0 otherwise

Goal: find n such that $\mathbb{P}(R < 0) \leq 1 - \alpha$, under the **solvency condition**

$$W + nQ_i \geq VaR_\alpha(X)$$

Example 2: Capital Requirement for a Multi-Year Policy

Consider a single multi-year policy over $m = 5$ years, with fixed indemnity and estimated adverse event probability $\hat{p} = 0.25$

What is the initial capital W required so that a premium loading of $c\%$ ensures a ruin probability no greater than $1 - \alpha$?

Remark The claim variable is defined to pay 1 whenever *at least one* of the two weather indicators deviates by one standard deviation from the reference range

$c = 0.1$	$c = 0.15$	$c = 0.2$
$1.625K$	$1.5625K$	$1.5K$

Reserve: $R = W + m \cdot Q - X$, where $X = KY$, with $Y \sim Bin(m, p)$, $Q = (1 + c)K\hat{p}$
Goal: find W such that $\mathbb{P}(R < 0) \leq 1 - \alpha$, under the **solvency condition**

$$W + m(1 + c)K\hat{p} \geq KVaR_{\alpha}(Y)$$

Example 3: Risk Pooling with Variable Indemnity

Consider one annual policy with variable indemnity and estimated probability of adverse event $\hat{p} = 0.25$

What is the initial capital W required so that a premium loading of c% ensures a ruin probability no greater than $1 - \alpha$?

	$c = 0.1$	$c = 0.15$	$c = 0.2$
$\alpha = 90\%$	0.412	0.408	0.402
$\alpha = 95\%$	0.863	0.858	0.853
$\alpha = 99\%$	1.702	1.697	1.692

Reserve: $R = W + \cdot Q - Y$, where

$$Y = \max\left(\frac{1}{\sigma_1}((\mu_1 - \sigma_1) - X_1)^+, \frac{1}{\sigma_2}((\mu_2 - \sigma_2) - X_2)^+\right)$$

Goal: find W such that $\mathbb{P}(R < 0) \leq 1 - \alpha$, under the **solvency condition**

$$W + (1 + c)K\hat{p} \geq VaR_\alpha(Y)$$

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In this Paper

- We propose a novel multi-index parametric insurance for agricultural weather risk
- We apply the proposal to real-world data: durum wheat (Bari province, Italy)
- We estimate the joint probability of extreme events and compute premium, payout, and other actuarial measures

Ongoing Work

- Compare estimation methods for adverse event probability (empirical, utility-based, etc.)
- Extend to stochastic models for weather variables and derivative-based portfolio strategies
- Explore new multivariate indices reflecting phenological traits and spatial-temporal patterns

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Thank you for your attention!

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