

# Excess Verdicts Insurance

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# Introduction to Excess Verdicts

- **Excess Verdicts (EVs)** aka “nuclear verdicts”: court-awarded damages far exceeding policy limits, often in cases like wrongful death or personal injury
- Unlike operational risks, EVs stem from **unpredictable legal decisions**, showing the limits of traditional risk models
- **Evolving Legal Factors:** *bad faith* and *negligence* can impact jury and judge decisions, increasing uncertainty and legal risks for insurers
- EVs lead to unexpected financial pressures, high legal costs for both policyholders and insurers, and compensation delays for injury party

# Contribution

- Study a **multi-environment** optimal insurance problem
  - ▶ optimal contract is **layer-type** (deductible and upper limit) in each environment
  - ▶ deductible is environment independent
- Application: ex-ante agreement in case of EV

# Ex Ante Agreements and Predictive Mechanisms

## Ex Ante Agreements:

Defines insurer and policyholder responsibilities in advance to reduce disputes and clarify financial distribution in excess verdict cases.

## Stage 1: Policy Limit Check

- **Trigger:** Court-awarded damages exceed policy limits.
- **Action:** Initiates Stage 2 review.

# Ex Ante Agreements and Predictive Mechanisms

## Stage 2: Insurer Conduct Review

- **Trigger:** Review of insurer's actions for good or bad faith.
- **Good Faith:** Insurer acts fairly and thoroughly, aiming to settle within limits.
- **Bad Faith:** Unreasonable actions like delaying claims or unfairly rejecting settlements.
- **Outcome:** Financial responsibility is allocated based on the insurer's conduct.

# Comparing Buyer and Seller Payments in EV Contracts

## Contract Payments With vs. Without Environment-Contingent Provisions

- **Goal:** Compare payments under contracts **with and without environment-contingent provisions** under EVs
- **Scenarios:**
  - ▶ **Y=1:** Damages within policy limits (no excess)
  - ▶ **Y=2:** Damages exceed limits, no insurer bad faith
  - ▶ **Y=3:** Excess verdict with insurer bad faith

# Comparing Buyer and Seller Payments in EV Contracts

Scenario	Party	Without Provisions	With Provisions
<b>Y=1</b>	Buyer	$\hat{R}(X)$	$R_1(X)$
	Seller	$\hat{I}(X)$	$I_1(X)$
<b>Y=2</b>	Buyer	$\hat{R}(L) + (X - L)$	$R_2(L) + (X - L)$
	Seller	$\hat{I}(L)$	$I_2(L)$
<b>Y=3</b>	Buyer	$\hat{R}^c(X)$	$R_3(\tilde{L}) + (X - \tilde{L})_+$
	Seller	$\hat{I}^c(X)$	$I_3(X) = X - R_3(\tilde{L}) - (X - \tilde{L})_+$



# Multiple Indemnity Environments

## Problem Setting

- **Setup:** one-period economy and risk sharing between buyer and seller; risk  $X \geq 0$
- **Risk Environments:** exogenous environment  $Y$  partitions the sample space into  $m + 1$  disjoint subsets;  
if  $Y = k$ , buyer transfers  $I_k(X)$  to seller,  
retains  $R_k(X) = X - I_k(X)$
- **Admissible profiles:**

$$\mathcal{I} := \{ \mathbf{I} = (I_1, \dots, I_m) : 0 \leq I_k \leq Id, \\ I_k \text{ and } R_k \text{ non-decreasing for all } k = 1, \dots, m \}.$$

# Multiple Indemnity Environments

## Problem Setting

- **Premium paid to seller:** Wang's premium principle,  $g$  (nondecreasing, concave) distortion function,  $Z \geq 0$  risk

$$P_g(Z) = \int_0^\infty g(S_Z(z)) dz$$

- **Buyer's Risk Positions:**  $\rho \geq 0$  loading rate

$$\mathbf{B}(\mathbf{I}) := \sum_{k=1}^m R_k(X) 1_{\{Y=k\}} + (1 + \rho) P_g \left( \sum_{k=1}^m I_k(X) 1_{\{Y=k\}} \right)$$

# Multiple Indemnity Environments (cont'd)

## Problem Setting

- **Buyer's Risk Measure:**  $\varphi$  translation invariant, monotonic

$$\begin{aligned} F_{\varphi}(\mathbf{I}) &= \varphi(\mathbf{B}(\mathbf{I})) \\ &= \varphi\left(\sum_{k=1}^m R_k(X) \mathbf{1}_{\{Y=k\}}\right) + (1 + \rho)P_g\left(\sum_{k=1}^m I_k(X) \mathbf{1}_{\{Y=k\}}\right) \end{aligned}$$

## Buyer's problem

$$\inf_{\mathbf{I} \in \mathcal{I}} F_{\varphi}(\mathbf{I})$$

solution to this problem is Pareto efficient!

# Multiple Indemnity Environments (cont'd)

## Problem Setting

- **Subset of admissible indemnity profiles:**

$$\mathcal{I}^* = \left\{ \mathbf{I} \in \mathcal{I} : \text{for each } k = 1, \dots, m, \right. \\ \left. I_k(x) = (x - d_k)_+ - (x - l_k)_+ \text{ for some } d_k \leq l_k \right\}$$

Each indemnity profile in  $\mathcal{I}^*$  features layer-type transfers where, in any exogenous environment, the indemnity is full insurance beyond a deductible  $d_k$  and up to an upper limit  $l_k$

# Main Theorem

## Main Theorem

Let  $\varphi = \text{VaR}_\alpha$  or  $\varphi = \text{CVaR}_\alpha$ . For any  $\rho \geq 0$  and  $\mathbf{I} \in \mathcal{I}$ , there exists  $\tilde{\mathbf{I}} \in \mathcal{I}^*$  st

$$F_\varphi(\tilde{\mathbf{I}}) \leq F_\varphi(\mathbf{I})$$

## Corollary (1)

$\tilde{\mathbf{I}} = (\tilde{l}_1, \dots, \tilde{l}_m)$  can be chosen so that the deductible in each environment coincide

$$d_1 = \dots = d_k = d$$

## Corollary (2)

$F_\varphi(\tilde{\mathbf{I}}) < F_\varphi(\mathbf{I})$  if  $\mathbf{I} \notin \mathcal{I}^*$

## Extension

### Main Theorem

The Theorem still holds if the buyer's risk position is

$$\mathbf{B}(\mathbf{I}) := \sum_{k=1}^m R_k(X) 1_{\{Y=k\}} + \sum_{k=1}^m P_{g_k} \left( I_k(X) 1_{\{Y=k\}} \right)$$

a different distortion function in each environment to obtain the premium  
Special case of interest is the **proportional hazard transform (PH)**:

$$g_k(z) = z^{\beta_k}, \quad 0 < \beta_k \leq 1$$

# Simulation Study

## Setting

- $m = 3$  risk environments;  $(X|Y = k) \sim \text{type II Pareto}$  for all  $k$

$k$	$\mathbb{P}(Y = k)$	$\lambda$	$\alpha$	$\mathbb{E}[X Y = k]$	$SD[X Y = k]$
1	60%	40	5	10	12.91
2	30%	200	3	100	173.21
3	10%	1,500	2.5	1,000	2236.07

- Risk increases from scenario 1 to 3; scenario 3  $\equiv$  excess verdicts
- Use  $\varphi = \text{CVaR}_{95\%}$ . To adjust premiums for large losses, we use the proportional hazard transform with distortion function  $g(z) = z^{\beta_k}$

## Simulation Results (1/3)

$\beta_1$	Risk environment $Y_1$		Risk environment $Y_2$		Risk environment $Y_3$	
	$F_X(d)$	$S_X(l_1)$	$F_X(d)$	$S_X(l_2)$	$F_X(d)$	$S_X(l_3)$
0.45	93.13%	0.72%	32.80%	0.43%	4.57%	4.31%
0.55	89.44%	0.21%	27.58%	0.43%	3.69%	4.31%
0.65	85.40%	0.03%	23.60%	0.43%	3.06%	4.31%
0.75	81.19%	0.00%	20.48%	0.43%	2.60%	4.31%
0.85	76.93%	0.00%	17.95%	0.43%	2.24%	4.31%
0.95	72.71%	0.00%	15.87%	0.43%	1.95%	4.31%

**Table:** CDF at the deductible and survival function at the upper limit, conditional on each scenario, for different values of  $\beta_1$ .



## Simulation Results (3/3)

$\beta_2$	Risk environment $Y_1$		Risk environment $Y_2$		Risk environment $Y_3$	
	$F_X(d)$	$S_X(l_1)$	$F_X(d)$	$S_X(l_2)$	$F_X(d)$	$S_X(l_3)$
0.45	90.69%	0.032%	29.12%	1.44%	3.94%	4.31%
0.55	85.40%	0.032%	23.60%	0.43%	3.06%	4.31%
0.65	79.93%	0.032%	19.67%	0.06%	2.48%	4.31%
0.75	74.51%	0.032%	16.72%	0.00%	2.07%	4.31%
0.85	69.28%	0.000%	14.41%	0.00%	1.75%	4.31%
0.95	64.32%	0.000%	12.57%	0.00%	1.51%	4.31%

**Table:** CDF at the deductible and survival function at the upper limit, conditional on each scenario, for different values of  $\beta_2$ .

## Simulation Results (3/3)

$\beta_3$	Risk environment $Y_1$		Risk environment $Y_2$		Risk environment $Y_3$	
	$F_X(d)$	$S_X(l_1)$	$F_X(d)$	$S_X(l_2)$	$F_X(d)$	$S_X(l_3)$
0.45	85.40%	0.032%	23.60%	0.43%	3.06%	4.31%
0.55	78.72%	0.032%	18.95%	0.43%	2.38%	1.28%
0.65	72.53%	0.000%	15.80%	0.43%	1.94%	0.19%
0.75	67.09%	0.000%	13.56%	0.43%	1.64%	0.006%
0.85	62.45%	0.000%	11.94%	0.43%	1.43%	0.00002%
0.95	58.57%	0.000%	10.73%	0.43%	1.27%	0.00%

**Table:** CDF at the deductible and survival function at the upper limit, conditional on each scenario, for different values of  $\beta_3$ .

**Thank you for your attention!**